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Investigating Thermal Deflection in a Finite Hollow Cylinder Using Quasi-Static Approach and Space-Time Fractional Heat Conduction Equation

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ABSTRACT

This study embarks on an exploration of the thermal deflection characteristics of finite hollow cylinders, employing the space-time fractional heat conduction equation within a quasi-static framework. Heat application is executed on the upper surface of the cylinder, whilst maintaining a zero-temperature condition on the remaining boundaries. Temperature distribution across the cylinder is determined using the integral transform technique, a method ensuring precision in the computation of thermal responses. The discourse on thermal deflection is grounded in the principles of fractional diffusion wave theory, a contemporary approach providing deeper insights into heat conduction dynamics. Numerical analyses are presented, illustrating transient and long-range interaction responses of the hollow cylinder under various diffusion scenarios, namely sub-diffusion, normal diffusion, and super-diffusion.

1. INTRODUCTION

Fractional calculus has recently garnered significant attention in various engineering disciplines, including applications in proportional-integral-derivative controllers, fluid mechanics, bio-mathematics, viscoelasticity, electrochemistry, and signal processing. This surge in interest has catalyzed research in non-integer calculus. The concept of fractional-order calculus, while intriguing, presents substantial challenges in understanding its physical interpretations. Podlubny [1] has contributed to this field with a discussion on the geometric and physical exposition of fractional integration. A notable advantage of fractional differential equations is their nonlocal property, which offers a broader scope of application compared to traditional methods.

Riemann-Liouville's introduction of fractional derivatives has been instrumental in the evolution of fractional calculus. This concept has been extensively applied in mathematical formulations, offering unique and advantageous approaches. The field of fractional calculus has witnessed considerable research, driven by the interest in various methods of defining and utilizing fractional order derivatives. The adoption of fractional theory is attributed to its ability to represent delayed reactions to physical stimuli observed in nature, a feature not encapsulated by the generalized theory of thermoelasticity, which assumes immediate responses to such stimuli.

Sherief et al. [2] advanced the fractional-order theory of thermoelasticity, marking a significant contribution in this field. Povstenko [3-7] conducted in-depth studies on fractional thermoelasticity, employing the quasi-static theory. Raslan [8] addressed a specific problem related to a thick plate with symmetric temperature distribution. The work of Khobragade and Deshmukh [9] successfully tackled the inverse thermoelastic problem, providing a thorough analysis of quasi-static thermal deflection in circular plates. Deshmukh et al. [10] focused on a thin circular plate containing a heat source, employing a quasi-static methodology to ascertain the thermal deflection. Further contributions in this domain include those by Warbhe et al. [11, 12], who explored various problems within fractional order thermoelasticity using a quasi-static approach. Tripathi et al. [13] examined fractional order thermoelastic deflection in thin circular plates with constant temperature distribution. Additionally, Tripathi et al. [14] solved a problem involving a heat source inducing a fractional order generalized thermoelastic response in a half space, which changed periodically. Recently, Warbhe [15] investigated simply supported rectangular plates, focusing on determining thermal stresses through thermal bending moments, facilitated by a time-dependent fractional derivative. Ezzat et al. [16, 17], El-Sayed and Gaber [18] delved into a range of problems in fractional-order thermoelasticity, expanding the scope of research in this specialized area.

Research on thermoelasticity in fractional-order space-time domains has attracted attention from various scholars. Fil' Shtinskii et al. [19] successfully solved the equation describing heat flow in fractional space and time, further analyzing its thermoelastic behavior in a one-dimensional half-space scenario. Povstenko [20] has contributed significantly to the discourse on space and time fractional diffusion equations. Sherief et al. [21] explored the realm of two-dimensional half-space problems, delving into the novel theory of fractional order thermoelasticity. Hussein [22] focused on fractional order thermoelastic problems in the context of an infinitely